Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **13 (2006)** 101-115

ON INTEGRO–DIFFERENTIAL EQUATIONS WITH DELAYED ARGUMENTS

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Abstract. The monotone technique is applied to delay differential problems with initial and periodic conditions. This paper deals with problems of existence and uniqueness of solutions for above equations. Integro–differential inequalities are also discussed. The presented examples, which may concern some engineering problems, show that the corresponding assumptions are satisfied.
Keywords. Monotone iterative method, lower and upper solutions, convergence, existence of solutions, uniqueness of solutions, problems with maxima and delayed arguments.
AMS (MOS) subject classification: 34A12, 34A45, 34K10

1 Introduction

Consider the problem

$$\begin{cases} x'(t) = f\left(t, x(t), \max\limits_{[0,\alpha(t)]} x(s), x(\beta(t)), \int_0^{\gamma(t)} x(s)ds\right) \equiv Fx(t),\ t \in J, \\ x(0) = k \in \mathbb{R} \end{cases} \tag{1}$$

for $J = [0, T]$, where $T > 0$ is a fixed number and

$H_1:\ f \in C(J \times \mathbb{R}^4, \mathbb{R})$,

$H_2:\ \alpha, \beta, \gamma \in C(J, J),\ 0 \le \alpha(t) \le t,\ 0 \le \beta(t) \le t,\ 0 \le \gamma(t) \le t.$

Equations with maxima may find applications in the theory of automatic regulation, control theory and nonlinear mechanics (see, for example, [10],[11],[12]). Note that equations with maxima generate functionals not having the property of linearity even if the equations are linear. Indeed, problems of type (1) are more general in comparison with equations discussed in [10], [11]. Special cases of (1) under the assumptions that F satisfies the Lipschitz condition (with respect to the last four variables) were considered, for example in [2]. In this paper we apply the monotone iterative technique, for details, see for example [5], [6]; see also [1], [3], [4], [7]–[9], [13]. We formulate existence results under weaker conditions assuming that F satisfies the