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ON GENERALIZED VECTOR VARIATIONAL INEQUALITIES WITH SET-VALUED MAPPINGS

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Abstract. In the present paper, we studied some new generalized vector variational inequalities with set-valued mappings and proved some new existence theorems of solutions for the generalized vector variational-type inequalities with set-valued mappings in Hausdorff topological vector spaces. Our results extend and improve some corresponding results in this field.
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1 Introduction and Preliminaries

The theory of variational inequality today is large and rapidly developing subject in many kinds of branches of mathematics and engineering sciences. First of all, we describe the general setting for our results.

Let X and Y be two Hausdorff topological vector spaces and let K be a nonempty compact convex subset of X. Suppose $T : K \to 2^{L(X,Y)}$, $\theta : K \times K \to X$, $\eta : K \times K \to Y$ and $G : K \to 2^K$ are four mappings, where $L(X,Y)$ denotes the topological vector space generated by all linear continuous operators from X into Y. Let $\{C(x) : x \in K\}$ be a family of closed, convex cones in Y such that $C(x) \neq Y$ and $intC(x) \neq \emptyset$ for all $x \in K$, where $intA$ denotes the interior of a set A.

A partial order $\leq_{C(x)}$ in Y is defined by $y_1 \leq_{C(x)} y_2$ if and only if $y_2 - y_1 \in C(x)$ for any $y_1, y_2 \in Y$.

Definition 1.1 A mapping $f : K \times K \to Y$ is said to be convex if for any $(x_1, y_1), (x_2, y_2) \in K \times K$ and $t \in (0, 1)$ such that

$$f(tx_1 + (1-t)x_2, ty_1 + (1-t)y_2) \leq_{C(x)} tf(x_1, y_1) + (1-t)f(x_2, y_2),$$

that is,

$$tf(x_1, y_1) + (1-t)f(x_2, y_2) - f(tx_1 + (1-t)x_2, ty_1 + (1-t)y_2) \in C(x).$$