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OPTIMAL NUMERICAL STRATEGY FOR NASH GAMES OF WEAKLY COUPLED LARGE–SCALE SYSTEMS

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Abstract. In this paper, the linear quadratic N–players Nash games for infinite horizon weakly coupled large–scale systems are discussed. The strategies are obtained by solving the coupled Riccati type equations (CRTE) via a new numerical algorithm. Firstly, the asymptotic expansions and the uniqueness of the CRTE in a neighborhood of the weakly coupled parameter $\varepsilon = 0$ are established. The main contribution in this paper is that the proposed algorithm which is based on the Lyapunov iterations attains the linear convergence. As a result, it is proved that the approximate Nash equilibrium strategies achieve the high–order approximation of the cost performance. Furthermore, when the weak coupling parameter is unknown, it is also shown that the proposed parameter independent strategies are equivalent to the classical linear quadratic approximate controllers.

1 Introduction

The linear quadratic Nash games and their applications have been studied widely in many literatures (see e.g., [1]). It is well–known that in order to obtain the Nash equilibrium strategy, the coupled Riccati type equations (CRTE) must be solved. The Newton–type algorithm for solving the CRTE has been applied [2]. However, this research has concentrated on determining feedback gain matrices for the 2–players Nash games. It should be noted that for the general N–players Nash games, it is hard to solve the N–coupled CRTE (see e.g., [3] and reference therein). That is, when the N–players Nash games are considered via the Newton's method, the required work space is needed to N times of the dimension of the full–systems. Recently, an algorithm which is called the Lyapunov iterations for solving the CRTE has been introduced [4]. Although the Lyapunov iterations can be computed in the same systems dimension, there are no results for the convergence rate of the Lyapunov iterations. In order to improve the convergence rate of the Lyapunov iterations, the Riccati iterations for solving the CRTE have been proposed [5, 13]. However, the convergence proof of these algorithms has not been shown.

The control problems of the large–scale systems have been investigated extensively (see e.g., [6]). Particularly, the control problems of the weakly coupled large–scale systems have been studied by several researchers (see