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CHAOTIC VIBRATIONS OF THE WAVE EQUATION DUE TO A VAN DER POL TYPE NONLINEAR BOUNDARY CONDITION[1]

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Abstract. Chaotic dynamics has been hailed as the third great scientific revolution of the 20th century, along with relativity and quantum mechanics [1]. In this paper, we consider the one-dimensional wave equation with the initial-boundary value problem on the unit interval. The boundary condition at the left endpoint is linear homogeneous, injecting energy into the system, while the boundary condition at the right endpoint has cubic nonlinearity of a van der Pol type. We formulate the problem into an equivalent first order hyperbolic system, and use the method of characteristics to derive a nonlinear reflection relation caused by the nonlinear boundary conditions. Since the solution of the first order hyperbolic system depends completely on this nonlinear relation and its iterates, the problems is reduced to discrete iteration problem of the type $u_{n+1} = F(u_n)$, where F is the nonlinear reflection relation. We show that the interactions of these linear and nonlinear boundary conditions do not cause period-doubling to the Riemann invariants $(u,\ v)$ of the wave equation. We further illustrate that when the initial data are smooth satisfying certain compatibility conditions at the boundary points, the state of the wave equation can be chaotic.

Keywords: wave equation, period-doubling, chaos, topological conjugacy, hyperbolic system, vibration.

AMS (MOS) Subject Classification: 35L05, 35B10, 74H45, 74H65.

1 Introduction

Many physical, chemical and biological phenomena can be described by linear or nonlinear models. The last few decades have seen an explosion of interest in the study of linear and nonlinear dynamical systems and in the development of related concepts. This has been driven by modern computer power as well as by the discovery of new mathematical techniques, which include two contrasting themes: (i) the theory of dynamical systems, most popularly associated with the study of chaos, and (ii) the theory of integrable systems associated, among other things, with the study of solitons.

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