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DYNAMICAL STATE AND CONTROL RECONSTRUCTION FOR A PHASE FIELD MODEL

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Abstract. The problem of reconstructing the state and control functions of a distributed parameter system from measurements of its observable part is considered for a phase field model. An algorithm is suggested that is stable with respect to measurement errors and computational errors. It is based on the ideas of the theory of feedback control.
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1 Introduction

In this paper, we consider the problem to reconstruct an unobservable part of the state of a nonlinear parabolic control system together with the associated unknown control from certain measurements. The reconstruction process is dynamical with respect to the time, i.e., it exploits information on the state that have been obtained up to the current instant of time.

We present our method for the following phase field model that has been introduced in [2, 3, 5, 7, 8],

$$\frac{\partial}{\partial t}\psi + l\frac{\partial}{\partial t}\varphi = \Delta\psi + u \quad \text{in} \quad \Omega \times (0, \vartheta], \tag{1.1}$$

$$\frac{\partial}{\partial t}\varphi = \Delta\varphi + g(\varphi) + \psi \quad \text{in} \quad \Omega \times (0, \vartheta], \tag{1.2}$$

with boundary conditions

$$\frac{\partial}{\partial n}\psi = \frac{\partial}{\partial n}\varphi = 0 \quad \text{on} \quad \partial\Omega \times (0, \vartheta], \tag{1.3}$$

and initial conditions

$$\psi(0) = \psi_0, \quad \varphi(0) = \varphi_0 \quad \text{in} \quad \Omega. \tag{1.4}$$

Here, $\Omega \subset \mathbb{R}^n$ is a bounded domain, the state functions are φ (phase function) and ψ (temperature), and the control function is u. All these functions