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## THE THEORY OF $S_{MAX}$ GRAPH DOES NOT SEEM TO WORK IN A REAL CASE STUDY

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Abstract. A recent study has shown that the power-law degree distribution alone may not guarantee a network be scale-free. Meanwhile, a structural metric was formulated to represent the extent to which a network is scale-free, where a new concept of  $s_{max}$  graph was suggested, associated with a measure of a network being fully scale-free. Following this approach, in our recent investigation of a real competitive relationship network of enterprises in the software industry in Guangzhou city, China, we constructed and analyzed the  $s_{max}$  graph of the real network, hoping that this case study could help us confirm the seemingly attractive new theory. It turns out, however, that the concept of  $s_{max}$  graph does not preserve many essential features of the real network. To that end, the main reason for the failure of the original concept of  $s_{max}$  graph is discussed, with a possible remedy suggested for future research.

**Keywords.** Scale-free network,  $s_{max}$  graph, competitive relationship network, power-law distribution

## 1 Introduction

Since the Barabási-Albert (BA) model proposed in 1999 [1], all networks following a power-law degree distribution have been thought of as scale-free networks. However, it was recently pointed out that some of such complex networks such as the Internet at the router level and the metabolic network do not have many claimed scale-free features [3,4], whereas a scale-free network is believed to be one that has the following properties: (i) has a power-law degree distribution, (ii) is possibly generated by a certain random process, (iii) has hub-like core(s), therefore is "robust yet fragile", (iv) can be preserved under random degree-preserving rewiring, (v) has self-similarity, and (vi) does not depend on domain-specific details. Consequently, a structural metric, denoted by S(g), was proposed as a metric to measure to what extent a network is scale-free. In doing so, a network having a power-law degree distribution along with a high S(g) value can be defined as a scale-free network, called an  $s_{max}$  graph [3,4].