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NEW OSCILLATION CRITERIA FOR CERTAIN NONLINEAR SYSTEM OF DIFFERENTIAL EQUATIONS[1]

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Abstract. New oscillation criteria are established for nonlinear system of differential equation, which are different from most known ones in the sense that they are base on the information only on a sequence of subinterval of $[t_0, \infty)$, rather than on the whole half-line. Our results are shaper than some previous results.

Keywords. Oscillation, Interval criteria, Nonlinear system.
AMS (MOS) subject classification: 34C10, 34A30.

1 Introduction

In this paper, we consider the nonlinear system of differential equation

$$u_1' = |u_2|^{\lambda_1}\operatorname{sgn} u_2 + b_1(t)u_1, \quad u_2' = |u_1|^{\lambda_2}\operatorname{sgn} u_1 - b_2(t)u_2, \tag{1}$$

where the functions $b_i(t)$ $(i = 1, 2)$ are nonnegative and summable on each finite segment of the interval $[t_0, \infty)$, $\lambda_i > 0$ $(i = 1, 2)$ with $\lambda_1\lambda_2 = 1$.

System (1) can be considered as a generalization of the Emden-Fowler differential equation

$$v_1' = c_1(s)|v_2|^{\lambda_1}\operatorname{sgn} v_2, \quad v_2' = c_2(s)|v_1|^{\lambda_2}\operatorname{sgn} v_1. \tag{2}$$

In fact, the change of variables

$$t = \int_0^s c_2(\tau)\left(\frac{c_2(\tau)}{c_1(\tau)}\right)^{-\frac{1+\lambda_2}{2+\lambda_1+\lambda_2}} \mathrm{d}\tau, v_i(s) = \left(\frac{c_i(s)}{c_{3-i}(s)}\right)^{\frac{1}{2+\lambda_1+\lambda_2}} u_i(t) \quad (i = 1, 2)$$

reduces system (2) to the nonlinear differential equation

$$u_1' = |u_2|^{\lambda_1}\operatorname{sgn} u_2 + b(t)u_1, \quad u_2' = |u_1|^{\lambda_2}\operatorname{sgn} u_1 - b(t)u_2,$$

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