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PROJECTIVE MODULI SPACE FOR THE POLYNOMIALS

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Abstract. The projective moduli space is defined as the space of all collections of conjugacy classes of polynomial maps that the sum of their degrees equals to n or less. In this paper, we define a map from the projective moduli space to $\mathbb{CP}^{n-1}$ via the elementary symmetric functions of the multipliers at the fixed points. For the case of degree 4, we give the transformation formula, and show that quartic polynomials degenerate into quadratic polynomials on the set called exceptional set.
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1 Introduction

The space of the polynomials of degree n is a smooth complex $(n+1)$-manifold. The moduli space, consisting of all affine conjugacy classes of the maps, is considered as a smooth complex $(n-1)$-manifold under some conditions. The projection Ψ_n from this moduli space to $\mathbb{C}^{n-1}$ is obtained via the elementary symmetric functions of the multipliers at the fixed points. In [2] (see also [4]), we proved the projection is not surjective for every $n \geqq 4$. The image of the moduli space under this projection Ψ_n is denoted by $\Sigma(n)$. The complement of $\Sigma(n)$ is denoted by $\mathcal{E}(n)$, and called the exceptional set. It is very interested to analyze the exceptional set $\mathcal{E}(n)$. For $n = 4$, $\mathcal{E}(4)$ forms the punctured curve [5]. And on a part of $\mathcal{E}(4)$, a quartic polynomial degenerate into "twins" of quadratic polynomials conjugate to z^2+c by using the method of polynomial-like mapping [3].

This paper is a part of series of our study [2] and extends it to polynomials of degree n or less. The projective moduli space is defined as the space of all collections of conjugacy classes of polynomial maps that the sum of their degrees equals to n or less. Since the exceptional set is nowhere dense (see [2]), a natural compactification of the set $\Sigma(n)$ is the projective space $\mathbb{CP}^{n-1}$. Then, we discuss the correspondence between two spaces: the projective moduli space and the compactification of $\Sigma(n)$.