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EXISTENCE RESULTS FOR A PARTIAL SECOND ORDER FUNCTIONAL DIFFERENTIAL EQUATION WITH IMPULSES

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Abstract. In this work we study the existence and regularity of mild solutions for a partial second order functional differential equation with impulses.

Keywords. Differential equations in abstract spaces, impulsive differential equations, cosine functions of operators, mild solutions, regularity of mild solutions.

AMS (MOS) subject classification: 34K30, 34K45, 47D09.

1 Introduction

In this paper we study the existence and regularity of mild solutions, a concept to be introduced later, for a class of partial second order functional differential equation with impulses described in the form

$$\begin{aligned} u''(t) &= Au(t) + f(t, u(t), u(a(t)), u'(t), u'(b(t))), \\ &\qquad t \in I = [0,T],\ t \neq t_i, i = 1, \cdots, n, \quad (1) \\ u(0) &= y_0, \quad u'(0) = y_1, \quad (2) \\ \triangle u(t_i) &= I_i(u(t_i), u'(t_i^-)), \qquad i = 1, \cdots, n, \quad (3) \\ \triangle u'(t_i) &= J_i(u(t_i), u'(t_i^-)), \qquad i = 1, \cdots, n, \quad (4) \end{aligned}$$

where A is the infinitesimal generator of a strongly continuous cosine function of bounded linear operators $(C(t))_{t\in\mathbb{R}}$ on a Banach space X; $y_0, y_1 \in X$; $0 < t_1 < \cdots < t_n < T$ are prefixed numbers; $a(\cdot)$, $b(\cdot)$, $I_i(\cdot), J_i(\cdot)$ and $f(\cdot)$ are appropriate functions and the symbol $\triangle\xi(t)$ represents the jump of the function $\xi(\cdot)$ at t, which is defined by $\triangle\xi(t) = \xi(t^+) - \xi(t^-)$.

The theory of impulsive differential equations has become an important area of investigation in recent years stimulated by their numerous applications to problems arising in mechanics, electrical engineering, medicine, biology, ecology, etc. Ordinary differential equations of first and second order with impulses have been treated in several works, see for example [11, 2, 1, 10, 6, 17] and the references therein. Partial differential equations with impulses are studied, for instance, by Liu [12], Rogovchenko [14, 15] and Hernández [8].