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EXISTENCE AND OSCILLATION RESULTS OF SOME IMPULSIVE DELAYED INTEGRODIFFERENTIAL PROBLEM

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Abstract. We prove in this paper that existence and nonoscillation of solutions of some impulsive delayed integrodifferential problem is equivalent to the existence and nonoscillation of solutions of some appropriate integral equation without impulses.

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AMS (MOS) subject classification: 34A37, 47D09, 34G20.

1 Introduction

In the recent years the successive developments of the theory of impulsive differential equation and delay differential equation have made it one of the main areas of Mathematics which attracts a great deal of new investigators. To learn more on the subject we refer to the book **[3]**, and regarding the delay differential equation, see **[2]**. In this paper, inspired by the paper of A. Zhao and J. Yan **[5]**, and themselves based on the work of K.Gopalsamy and B. G. Zhang **[1]**, we shall generalize some results in **[5]** by introducing an integral term into the differential equation which makes the solution $x(t)$ depend on the states preceding the time t.
We shall consider the following integrodifferential delay equation

$$x^{'}(t) + \sum_{i=1}^{n} a_i(t)\, x(t-\tau_i) + \int_{\sigma}^{t} g(t,s)\, x(s)\, ds = 0, \qquad t \neq t_k \tag{1}$$

subject to the impulsive conditions

$$x\left(t_k^{+}\right) - x(t_k) = b_k x(t_k), \qquad k = 1, 2, ... \tag{2}$$

We establish some nonoscillation results for the solution of problem **(1)-(2)** which are based on some correspondence between the existence of positive solutions of this problem and solutions of certain integral equation. In **[5]** the authors studied the problem without the integral term, while in **[1]** the authors considered only one time delay τ and no integral term either.
Let us set the following assumptions
$\mathbf{A}_1$) $0 < t_1 < t_2 < ... < t_k < ...$ and $\lim_{k\to\infty} t_k = \infty$;