A CLASS OF SEMILINEAR PARABOLIC AND HYPERBOLIC SYSTEMS DETERMINED BY OPERATOR VALUED MEASURES

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Abstract. In this paper we consider a class of semilinear and strongly nonlinear distributed parameter systems governed by parabolic and hyperbolic partial differential equations containing operators which are measures. We prove existence, uniqueness and regularity properties of weak solutions. This extends our recent results on linear systems covering the important and well established class of systems considered by J.L Lions as special case. Some illustrating examples are provided from strongly nonlinear parabolic and semilinear hyperbolic problems.

Keywords. Parabolic and hyperbolic systems, Coercive operators, Operator valued measures, Existence of Solutions, Regularity properties.

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1. Introduction

In three recent papers [1,2,3] the author studied systems governed by vector and operator valued measures under different assumptions with increasing generality. In [3] abstract linear parabolic and hyperbolic partial differential equations were considered where the principal operator itself is a measure which was assumed to be countably additive in the weak operator topology. We used the classical Galerkin technique to establish existence and uniqueness of weak solutions including the regularity properties. Our objective here is to extend some of the results of [3] from linear to semilinear and strongly nonlinear problems. These results complement and extend the results found in [4-8,10].

The rest of the paper is organized as follows. In section 2 some notations and terminologies are presented. In section 3 linear parabolic problems driven by measures are considered. Questions of existence of weak solutions and their regularity properties are studied. In section 4, similar studies are made for semilinear and strongly nonlinear parabolic problems driven by measures.