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SCATTERING OF SMALL AMPLITUDE SOLUTIONS FOR A MULTIDIMENSIONAL GENERALIZED BOUSSINESQ EQUATION

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Abstract. In this paper, we consider the Cauchy problem for a class of the multidimensional generalized Boussinesq equation. And the global existence and nonlinear scattering for small amplitude solutions are established.

Keywords. Boussinesq equation; Decay estimates; Cauchy problem; Existence of the global solution; Scatter.

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1 Introduction

In this paper, we study the global existence of small amplitude solutions for the following Cauchy problem of the generalized Boussinesq equation

$$u_{tt} - \Delta u_{tt} + \Delta^2 u_{tt} = -\Delta^2 u + \Delta u + \Delta f(u), (x,t) \in R^n \times (0,+\infty), \quad (1)$$
$$u(x,0) = \phi(x), u_t(x,0) = \psi(x), x \in R^n, \quad (2)$$

$\phi(x), \psi(x)$ are the initial value functions, $f(u)$ are the nonlinear functions.

Recently, in order to investigate the water wave problem with surface tension, Schneider [14] consider a class of Boussinesq equation which models the water wave problem with surface tension as follows:

$$u_{tt} = u_{xx} + u_{xxtt} + \mu u_{xxxx} - u_{xxxxtt} + (u^2)_{xx}, \quad (3)$$

where $x, t, \mu \in R$ and $u(t,x) \in R$. The model can also be formally derived from the 2D water wave problem. For a degenerate case, Schneider proved that the long wave limit can be described approximately by two decoupled Kawahara-equations. A more natural model might seem to be extension of the classical Boussinesq equation as follows (see [2]):

$$u_{tt} = u_{xx} + (\mu + 1)u_{xxxx} - u_{xxxxxx} + (u^2)_{xx}. \quad (4)$$

In [14], the author included the term with the sixth order derivatives since he was interested precisely in the case when the coefficient in front of the