Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **14 (2007)** 497-507
 http://www.watam.org

GENERALIZED QUASILINEARIZATION FOR PERIODIC PROBLEMS

Rahmat Ali Khan

Department of Mathematics
University of Glasgow, Glasgow G12 8QW, UK

Abstract. We study the upper and lower solutions method and the generalized method of quasilinearization for the solutions of some second order periodic boundary value problems in the presence of an upper solution β and a lower solution α in the reversed ordered ($\alpha \geq \beta$). We also discuss quadratic convergence of the sequence of approximants.
Keywords. Periodic problems, upper and lower solutions, Quasilinearization, Quadratic Convergence

1 Introduction

In this paper, we consider some second order nonlinear periodic problems of the type

$$\begin{aligned} -x''(t) &= f(t, x(t)),\ t \in [0, T], \\ x(0) &= x(T),\ x'(0) = x'(T), \end{aligned} \tag{1.1}$$

where $f : [0, T] \times \mathbb{R} \to \mathbb{R}$ is continuous. We study existence and approximation of solutions in the presence of lower solution α and an upper solution β with the reversed ordered ($\alpha \geq \beta$ on $[0, T]$). We study the method of upper and lower solutions and develop the quasilinearization technique for the existence and approximation of solutions. We show that under suitable conditions the sequence of approximants obtained by the method of quasilinearization converges quadratically to a solution of the original problem.
There is a vast literature dealing with the solvability of nonlinear boundary value problems with the method of upper and lower solution and the quasilinearization technique in the case where the lower solution α and the upper solution β are ordered by $\alpha \leq \beta$. Recently, the case where the upper and lower solutions are in the reversed order has also received some attention. A. Cabada, et al. [4, 5], M. Cherpion, et al. [6] have studied existence results for Neumann problems in the presence of lower and upper solutions in the reversed ordered. In these papers, they developed monotone iterative technique for existence of a solution x such that $\alpha \geq x \geq \beta$.
The main idea of the method of quasilinearization developed by Bellman