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TWO-STEP MACCORMACK METHOD FOR STATISTICAL MOMENTS OF A STOCHASTIC BURGER'S EQUATION

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Abstract. The two-step MacCormack scheme has been modified to solve a stochastic Burger's equation driven by a random force with a random initial condition. Statistical moments of a solution are expressed by Hermite-Fourier coefficients so that the stochastic equation is transformed into a deterministic propagator system. The resultant system needs to be solved only once and computational loads are reduced accordingly. The numerical stability, accuracy and efficiency of the scheme have been analyzed and compared with the Monte Carlo method and the Lax-Wendroff scheme. The modified MacCormack scheme shows less diffusion near discontinuities than the Lax-Wendroff scheme. While maintaining the same accuracy, the MacCormack scheme improves numerical efficiency over the Lax-Wendroff scheme in the ratio of $(N + 11/6)$ when the length is N. Compared to the Monte Carlo method, the scheme saves more than 98% of CPU time and removes dependence upon a random number generator.

Keywords. MacCormack scheme, Stochastic differential equation, Burger's equation, Lax-Wendroff scheme, Monte Carlo method

AMS (MOS) subject classification: 65M06,60H15;42C15,65C05

1 Introduction

It is common to view uncertainty in mathematical models such as Euler equations for gas dynamics, the Buckley-Leverett equation for multi-phase flow in porous media [9, 10, 11] or the Navier-Stokes equation [16, 23]. This may result from insufficient information such as imprecise coefficient or input values [28]. Limited understanding of underlying phenomena may cause such a noise [13]. Observational errors due to inaccuracies in the measurements may not be removed and lead to a filtering problem [28, 32]. Black-Scholes model for option pricing in finance [18] is another example for uncertainties. In order to estimate these effects, uncertainties are mathematically modeled by random fields. Then, the governing equation is represented in the form of a stochastic partial differential equation (SPDE). Since the solution of this type of equation is random, one particular solution corresponding to a specific realization is not of concern. Instead, it is important to know statistical properties of the solution such as its mean or variance.