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GENERATION AND CHARACTERIZATION OF NONLINEAR SEMIGROUPS ASSOCIATED TO SEMILINEAR EVOLUTION EQUATIONS INVOLVING "GENERALIZED" DISSIPATIVE OPERATORS

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Abstract. Given a linear operator A which satisfies a generalized dissipativity condition in terms of a "uniqueness function" w and its nonlinear continuous perturbation B in a real Banach space X, we discuss the construction of a nonlinear semigroup S providing mild solutions for the semilinear abstract Cauchy problem (SP) $u' = (A + B)u(t), t > 0$ and $u(0) = x$. It is shown that a subtangential condition and a semilinear stability condition are altogether necessary and sufficient for the generation of the semigroup S. A concrete example to which these generation results are applicable is also provided.
Keywords. "generalized" dissipative operators, nonlinear semigroups, generation theorems, semilinear evolution equations, semilinear stability condition, discrete schemes.
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1 Introduction

Since the fundamental paper [1] of Crandall and Liggett has been published, generation theory for nonlinear semigroups on arbitrary Banach spaces has evolved into a well-established subject, being used to treat a broad class of mathematical models due to its considerable unifying effect. A significant improvement of their theory has been made by Kobayashi in [6], who replaced the range condition used in [1] with a much less restrictive assumption, called the tangency range condition. Further, in their later paper [7], Kobayashi and Tanaka also succeeded in weakening Crandall and Liggett's classical dissipativity condition to a more general assumption of dissipativity with respect to a so-called uniqueness function, therefore opening the way for

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