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HOPF BIFURCATION OF A NONLINEAR LASOTA-WAZEWSKA-TYPE POPULATION MODEL WITH MATURATION DELAY

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Abstract. In this paper, the stability and Hopf bifurcation analysis of a nonlinear Lasota-Wazewska-type population model with maturation delay are presented. It shows that the system may exhibit stability switches as the delay crosses the critical values of delay, where a Hopf bifurcation may occur. On the basis of the newly developed method of pseudo-energy that involves easy computation only, the local dynamics near a Hopf bifurcation is determined by that of the generated pseudo-vibration system. The numerical simulation is in very good agreement with the theoretical prediction.
Keywords. time delay, stability switches, Hopf bifurcation, pseudo-energy analysis.
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1 Introduction

Since Malthus's population theory was presented, great progress has been achieved in the population dynamics [1]. The evolution of population species can be modelled by ordinary differential equations (ODEs for short), partial differential equations (PDEs), delay differential equations (DDEs) and so on [1]-[6]. The DDE models have been received much attention over the past decades [1][3]-[6]. This paper addresses on the nonlinear Lasota-Wazewska-type model with maturation delay for a single specie, described by

$$\dot{x}(t) = -\mu\, x(t) + p\, \mathrm{e}^{-d\,\tau}\, \mathrm{e}^{-\gamma\, x(t-\tau)} \tag{1}$$

where $x(t)$ stands for the total population of the single species at time t, $\tau > 0$ is the maturation delay, $\mu > 0$ is the death rate, $p > \mu$ and $\gamma > 0$ are two parameters in the birth function, d is the death rate constant for each life stage prior to the adult stage [3]. The DDE model (1) with $d = 0$ is originally proposed for the red blood cells system [2] and has been investigated by some authors. In [4], for example, on the basis of the center manifold reduction, the local dynamics near a Hopf bifurcation has been investigated, and the

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