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GLOBAL L^n STRONG SOLUTIONS TO MAGNETO-HYDRO-DYNAMICS EQUATIONS IN THE $\mathbb{R}^n$ SPACE

Chunshan Zhao [1] and Yinnian He [2]

[1]Department of Mathematical Sciences, Georgia Southern University
Statesboro, GA 30460, USA. Email: czhao@GeorgiaSouthern.edu
[2]Faculty of Science
Xi'an Jiaotong University, Xi'an, Shaanxi, 710049, P. R. China. Email:
heyn@mail.xjtu.edu.cn

Abstract. We study the existence and uniqueness of global L^n strong solutions to the Magneto-Hydro-Dynamics (MHD) equations in the whole $\mathbb{R}^n$ space. Under smallness assumption on suitable norms of initial data and external force, existence and uniqueness of global L^n strong solutions are proved. Moreover, we also present some algebraic decay properties of the unique global L^n strong solution under some assumptions on both initial data and external force.

Keywords. Strong solution, MHD equations, Algebraic decay, Existence and uniqueness.
AMS (MOS) subject classification: 35K55, 76D03, 35K45

1 Introduction

Let $Q = \mathbb{R}^n \times (0,\infty)(n \geq 2)$, we consider the Magneto-Hydro-Dynamics (MHD) equations [4] in Q as follows.

$$\begin{cases} \dfrac{\partial u}{\partial t} - \nu\Delta u + (u\cdot\nabla)u - \dfrac{1}{\rho\mu}(B\cdot\nabla)B \\ \quad + \dfrac{1}{2\rho\mu}\nabla(|B|^2) + \dfrac{1}{\rho}\nabla\Pi = f(x,t), \quad \text{in} \quad Q, \\ \dfrac{\partial B}{\partial t} - \lambda\Delta B + (u\cdot\nabla)B - (B\cdot\nabla)u = 0, \quad \text{in} \quad Q, \\ \nabla\cdot u = 0, \nabla\cdot B = 0, \quad \text{in} \quad Q, \\ \lim\limits_{|x|\to\infty} u = 0, \quad \lim\limits_{|x|\to\infty} B = 0, \quad \text{for} \quad t \in (0,\infty), \end{cases} \tag{1}$$

with initial data $u(x,0) = \tilde{u}_0(x)$ and $B(x,0) = \tilde{B}_0(x)$ satisfying $\nabla\cdot\tilde{u}_0(x) = \nabla\cdot\tilde{B}_0(x) = 0$ respectively. In (1), $u = (u^1(x,t),\cdots,u^n(x,t))$ and $B = (B^1(x,t),\cdots,B^n(x,t))$ are unknown velocity vector and magnetic field respectively. Π is pressure and $f(x,t)$ is external force. ν, μ and ρ are constants of kinematic viscosity, magnetic permeability and density of Eulerian flow respectively. $\lambda = \frac{\eta}{\mu}$ with electrical resistivity η.

In this paper, we shall show the existence and uniqueness of global L^n strong solutions to (1) under smallness assumption on suitable norms of both