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A NOTE ON BIFURCATION STRUCTURE OF RADIALLY SYMMETRIC STATIONARY SOLUTIONS FOR A REACTION-DIFFUSION SYSTEM

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Abstract. In this paper, we discuss the bifurcation structure of radially symmetric stationary solutions for a certain reaction-diffusion system. To do this, we shall employ the comparison principle and the numerical verification method, and estimate the integral for the Bessel function of the first kind which is an eigenfunction of the linearized operator around the constant solution.

Keywords. bifurcation structure, interval arithmetic, Bessel function

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1 Introduction

To understand the mechanism of phenomena which appear in various fields, we often use the system of reaction-diffusion equations

$$\begin{cases} \frac{\partial}{\partial t}\mathbf{u} = \varepsilon\, D\, \Delta \mathbf{u} + \mathbf{f}(\mathbf{u}), \qquad x \in \Omega, \qquad t > 0, \\ \frac{\partial}{\partial \nu}\mathbf{u} = \mathbf{0}, \qquad x \in \partial\Omega, \qquad t > 0 \end{cases} \tag{1.1}$$

with suitable initial condition, and discuss the existence and stability of stationary solutions for the system, where $\mathbf{u} \in \mathbb{R}^N$, $\varepsilon > 0$, D is a diagonal matrix whose elements are positive, $\mathbf{f} : \mathbb{R}^N \to \mathbb{R}^N$ is a smooth function in $\mathbf{u}$, Ω is a bounded domain in $\mathbb{R}^\ell$ with smooth boundary $\partial\Omega$, and $\frac{\partial}{\partial \nu}$ denotes the outward normal derivative on $\partial\Omega$.

When $N = 1$ is satisfied, we employ the so-called *comparison principle*, and study the existence of stationary solutions of (1.1) and their stability property. Moreover it is well-known that for suitable $\mathbf{f}(\mathbf{u})$, the global attractor $\mathcal{A}$ of (1.1) is represented as $\mathcal{A} = \bigcup_{\mathbf{e}\in E} W^u(\mathbf{e})$, where E is the set of stationary solutions of (1.1), and $W^u(\mathbf{e})$ is an unstable manifold of (1.1) at $\mathbf{u} = \mathbf{e}$ (for example, see Chapter 4 in Hale [2]). This result suggests that we need to seek out all stationary solutions of (1.1) in order to understand the precise asymptotic behavior of solutions for (1.1). Along this line, in this paper, we assume that there exists a solution $\hat{\mathbf{u}} \in \mathbb{R}^N$ of $\mathbf{f}(\mathbf{u}) = \mathbf{0}$ with $\det \mathbf{f}_{\mathbf{u}}(\mathbf{u}) < 0$, and we try to discuss the bifurcation structure of radially