OSCILLATION CRITERIA FOR FORCED NEUTRAL DIFFERENCE EQUATION WITH POSITIVE AND NEGATIVE COEFFICIENTS

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Abstract. In this paper, we provide oscillation properties of every solution of the forced neutral difference equation with positive and negative coefficients

$$\Delta(x_n - c_n x_{n-r}) + p_n x_{n-k} - q_n x_{n-l} = f_n,$$

where $\{p_n\}, \{q_n\}, \{c_n\}$ are sequences of nonnegative real numbers, $\{f_n\}$ is sequence of real numbers, k, l and r are integers with $0 \le l \le k - 1$, $r > 0$.

Keywords. Difference equation, forced, neutral, oscillation, positive and negative coefficients.

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1 Introduction

In this paper, we are dealing with oscillatory behavior of all solutions of the following forced neutral difference equation with positive and negative coefficients

$$\Delta(x_n - c_n x_{n-r}) + p_n x_{n-k} - q_n x_{n-l} = f_n, \quad n = 0, 1, 2, ..., \qquad (1)$$

where p_n, q_n, c_n are real numbers with $p_n \ge 0, q_n \ge 0, c_n \ge 0$, $\{f_n\}$ is sequence of real numbers, k, l and r are integers with $0 \le l \le k - 1$, $r > 0$. Further, here Δ denotes the forward difference operator by $\Delta x_n = x_{n+1} - x_n$. When $f_n \equiv 0$, equation (1) becomes

$$\Delta(x_n - c_n x_{n-r}) + p_n x_{n-k} - q_n x_{n-l} = 0, \quad n = 0, 1, 2, ..., \qquad (2)$$

The oscillation of every solution of equation (2) have been studied by some authors in $[1 - 9]$. In particular Ladas [2] and Qian and Ladas [6] considered the case where $c_n \equiv 0$. Chen and Zhang [1] and Zhang and Wang [9] considered the case where $c_n \equiv c$ with $0 \le c < 1$. Li and Cheng [3, 4] and Tang et al. [7] and Tian and Cheng [8] studied oscillatory behaviors of all solutions of (2). Further, Luo et al. [5] concerned with differential equation form of (1).