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OSCILLATION CRITERIA FOR LINEAR HAMILTONIAN SYSTEMS [1]

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Abstract– Using a generalized Riccati transformation and monotone functionals on suitable matrix space, some new oscillation criteria for linear matrix Hamiltonian systems are established. These results can be considered as generalizations and improvements of the results due to F. Meng and A. B. Mingarelli [2], I. S. Kumari and S. Umanaheswaram [4], and Q. Yang, R. Mathsen and S. Zhu [5].

Keywords– Linear Hamiltonian system, oscillation, monotone functional.

AMS (MOS) subject classification–34A30, 34C10.

1 Introduction

We consider oscillatory properties for the linear Hamiltonian vector system

$$\begin{cases} x' &= A(t)x + B(t)u, \\ u' &= C(t)x - A^*(t)u, \quad t \geq t_0, \end{cases} \tag{1.1}$$

where $A(t), B(t), C(t)$ are real $n \times n$ matrix-valued functions, B, C are Hermitian, B is positive definite. By M^* we mean the conjugate transpose of the matrix M. For any $n \times n$ Hermitian matrix M, its eigenvalues are real numbers, we always denote them by $\lambda_1[M] \geq \lambda_2[M] \geq \cdots \geq \lambda_n[M]$. The trace of M is denoted by $\mathrm{tr(M)}$ and $\mathrm{tr(M)} = \sum_{\mathrm{k=1}}^{\mathrm{n}} \lambda_{\mathrm{k}}(\mathrm{M})$.

We also consider the corresponding matrix system

$$\begin{cases} X' &= A(t)X + B(t)U, \\ U' &= C(t)X - A^*(t)U, \quad t \geq t_0. \end{cases} \tag{1.2}$$

For any two solutions $(X_1(t), U_1(t))$ and $(X_2(t), U_2(t))$ of system (1.2), the Wronskian matrix $X_1^*(t)U_2(t) - U_1^*(t)X_2(t)$ is a constant matrix. In particular, for any solution $(X(t), U(t))$ of system (1.2), $X^*(t)U(t) - U^*(t)X(t)$ is a constant matrix.

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