THE INITIAL VALUE AND THE PERIODIC PROBLEMS FOR A CLASS OF REACTION-DIFFUSION SYSTEMS

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Abstract. In this paper we prove an existence result for mild solutions to a reaction diffusion system of the form

$$\begin{cases} u'(t) = Au(t) + F(u(t), v(t)), & t \in \mathbb{R}_+ \\ v'(t) = Bv(t) + G(u(t), v(t)), & t \in \mathbb{R}_+ \\ u(0) = u_0 \\ v(0) = v_0, \end{cases}$$

where $(X, \|\cdot\|_X)$ and $(Y, \|\cdot\|_Y)$ are real Banach spaces, A, B are m-dissipative operators on X and respectively on Y, B generates a compact semigroup of contractions, $F : X \times Y \to X$ is continuous on $X \times Y$ and locally Lipschitz with respect to the first argument, while $G : X \times Y \to Y$ merely continuous. An existence result for periodic solutions as well as an example are also included.

Keywords. m-dissipative operator, compact semigroup, reaction-diffusion system, initial-value problem, periodic solution.

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1 Introduction

The purpose of this paper is to prove a local existence result concerning mild solutions to nonlinear reaction diffusion system of the form

$$\begin{cases} u'(t) = Au(t) + F(u(t), v(t)), & t \in \mathbb{R}_+ \\ v'(t) = Bv(t) + G(u(t), v(t)), & t \in \mathbb{R}_+ \\ u(0) = u_0 \\ v(0) = v_0. \end{cases} \tag{1.1}$$

where $(X, \|\cdot\|_X)$ and $(Y, \|\cdot\|_Y)$ are real Banach spaces and A, B, F, G satisfy the hypotheses:

(H_1) $A : D(A) \subset X \to X$ and $B : D(B) \subset Y \to Y$ are m-dissipative, possible nonlinear operators,

(H_2) B generates a compact semigroup on $\overline{D(B)}$,