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EXISTENCE OF HOMOCLINIC SOLUTIONS TO A NONLINEAR SECOND ORDER ODE

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Abstract. In this paper we are concerned with the existence of homoclinic solutions to Eq. (1.1) below. To this purpose, a classical method based on differential inequalities is used.

Keywords. Homoclinic solutions, differential inequalities.

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1. Introduction

This note is devoted to the existence of homoclinic solutions to the equation

$$x'' + 2f(t)\,x' + x + g(t,x) = 0, \qquad t \in \mathbb{R}, \tag{1.1}$$

i.e. solutions $x : \mathbb{R} \to \mathbb{R}$ satisfying the boundary conditions

$$x(\pm\infty) = x'(\pm\infty) = 0, \tag{1.2}$$

where $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ are two given functions, and

$$x(\pm\infty) := \lim_{t\to\pm\infty} x(t), \; x'(\pm\infty) := \lim_{t\to\pm\infty} x'(t).$$

We remark that problem (1.1) – (1.2) is closely related to the so-called *convergent solutions,* i.e. the solutions defined on $\mathbb{R}$ (or $\mathbb{R}_+ := [0, +\infty)$) and having finite limits to $\pm\infty$ (respectively $+\infty$) (see, e.g., [15], [20], [21]).

Problem (1.1) – (1.2) can also be considered a generalization of the periodic problem (1.1) – (1.3), where

$$x(a) = x(b), \; \dot{x}(a) = \dot{x}(b), \tag{1.3}$$

as $a \to -\infty$ and $b \to +\infty$.

The method used in this paper has a certain degree of generality, that allows it to be applied to several types of problems. It is based on Lemma 2.2 below. The function h which must be determined is the solution to a scalar differential equation, often called *equation of comparison*, the form of which