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APPLICATION OF DYNAMIC PROGRAMMING TO ECONOMIC PROBLEMS WITH VINTAGE CAPITAL

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Abstract. This is the second of two papers on boundary optimal control problems with linear state equation and convex cost arising in Economics and the the associated Hamilton–Jacobi–Bellman equation. In the first paper we studied existence and uniqueness of the solution of HJB in *strong* sense, namely the pointwise limit of classical solutions of approximating equations, which proves to be also Lipschitz in time and regular in the state variable. In this second paper we apply Dynamic Programming to show that the value function of the economic problem is the unique strong solution of the associated HJB equation.

Keywords. Linear convex control, boundary control, Hamilton–Jacobi–Bellman equations, optimal investment problems.

AMS (MOS) subject classification: 49J15, 49J20, 35B37

1 Introduction

This is the second of two papers regarding linear convex control in Hilbert spaces modeling boundary control for PDEs, and the Hamilton–Jacobi–Bellman (briefly, HJB) equation associated to the problem through Dynamic Programming. In the first paper [30] we established existence and uniqueness results for such a HJB equation, showing that there exists a unique *strong* solution (where by strong we mean, roughly speaking, the pointwise limit of classical solutions of approximating equations) which proves to be Lipschitz in time and C^1 in the state variable (the reader is referred to Definition 4.4, Theorem 4.5 for a review).

Here we discuss Dynamic Programming for the optimal control problem. More precisely, we let H and U be separable real Hilbert spaces with scalar products $(\cdot \mid \cdot)_H$ and $(\cdot \mid \cdot)_U$ respectively and we consider a dynamical system of the following type

$$\begin{cases} k'(\tau) = A_0 k(\tau) + Bu(\tau), & \tau \in]t, T[\\ k(t) = x \in H, \end{cases} \tag{1}$$

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