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ASYMPTOTIC PROPERTIES OF STOCHASTIC POPULATION DYNAMICS

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Abstract. In this paper we stochastically perturb the classical Lotka–Volterra model

$$\dot{x}(t) = \mathrm{diag}(x_1(t), \cdots, x_n(t))[b + Ax(t)]$$

into the stochastic differential equation

$$dx(t) = \mathrm{diag}(x_1(t), \cdots, x_n(t))[(b + Ax(t))dt + \beta dw(t)].$$

The main aim is to study the asymptotic properties of the solution. It is known (see e.g. [3, 20]) if the noise is too large then the population may become extinct with probability one. Our main aim here is to find out what happens if the noise is relatively small. In this paper we will establish some new asymptotic properties for the moments as well as for the sample paths of the solution. In particular, we will discuss the limit of the average in time of the sample paths.

Keywords. Brownian motion, stochastic differential equation, Itô's formula, average in time, boundedness.

AMS (MOS) subject classification: 60J65, 60H10, 34K40.

1 Introduction

The classical Lotka–Volterra model for n interacting species is described by the n-dimensional differential equation

$$\frac{dx(t)}{dt} = \mathrm{diag}(x_1(t), \cdots, x_n(t))[b + Ax(t)], \tag{1.1}$$

where

$$x = (x_1, \cdots, x_n)^T, \quad b = (b_1, \cdots, b_n)^T, \quad A = (a_{ij})_{n\times n}.$$

There is an extensive literature concerned with the dynamics of this model and we here only mention Ahmad and Rao [1], Bereketoglu and Gyori [4], Freedman and Ruan [9], He and Gopalsamy [11], Kuang and Smith [15], Teng