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ON PERIODIC SOLUTIONS OF LINEAR IMPULSIVE DELAY DIFFERENTIAL SYSTEMS[1]

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Abstract. A necessary and sufficient condition is established for the existence of periodic solutions of linear impulsive delay differential systems.

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1 Introduction

It is well known that (see eg., [20]) the nonhomogeneous linear system

$$x'(t) = A(t)x(t) + f(t) \tag{1}$$

has periodic solutions if and only if

$$\int_0^\omega y^T(t)f(t)\,\mathrm{d}t = 0 \tag{2}$$

for all periodic solutions $y(t)$ of period ω of the adjoint system

$$y'(t) = -A^T(t)y(t), \tag{3}$$

where $A \in C(\mathbb{R}, \mathbb{R}^{n\times n})$ and $f \in C(\mathbb{R}, \mathbb{R}^n)$ are periodic functions of period ω.

This result was extended in [8] to delay differential systems of the form

$$x'(t) = A(t)x(t) + B(t)x(t-\tau) + f(t), \tag{4}$$

where $A, B \in C(\mathbb{R}, \mathbb{R}^{n\times n})$ and $f \in C(\mathbb{R}, \mathbb{R}^n)$ are periodic functions of period ω and $\tau > 0$ is a fixed real number. Indeed, Halanay proved that (4) has

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