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POROUS MEDIUM DYNAMICS WITH OPTIMAL CONTROL AND EXTRACTION OF UNDERGROUND RESOURCES

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Abstract. In this paper we consider some mathematical models for flow of liquids or gas through porous media. We present some results on existence and uniqueness of solutions and their regularity properties. This is then applied to aquifers and a control problem is formulated. After presenting an existence theorem for optimal control policies, necessary conditions for optimality are developed.

Keywords. Mathematical Models, Porous Medium Equations, Nonlinear Partial differential equations, Aquifers, Optimal control.

AMS (MOS) subject classification: 34A36,34A60,49J24,93C10.

1. Introduction

In the study of hydrological systems, in particular reservoir engineering, the dynamics of flow through porous medium such as soil composed of sand and gravels play an important role in the design of extraction programs of water resources from aquifers. In particular, aquifers located near the seacoast suffer from contamination by salt water through seepage. Other applications include movement of pollutants from landfills through the soil system, including advanced methods of oil recovery and underground gas storage. The methodology developed here is generally applicable to mining of underground resources like gas, oil, water etc.

The rest of the paper is organized as follows. In section 2 the basic mathematical model of flow through porous medium described by a class of nonlinear partial differential equations is presented. Existence and regularity properties of solutions are discussed. In section 3, a control problem for resource extraction is formulated taking into account relevant physical constraints. In section 4 and 5, existence of optimal controls and necessary conditions of optimality are presented. Some extensions are presented in section 6 proving the existence of optimal controls. The paper is concluded with