NEW CONCEPTS FOR SEQUENCES AND DISCRETE SYSTEMS (II)

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Abstract. Based on our previous research, several new concepts for real sequences and discrete systems are given by means of frequency measure (also called asymptotic density or natural density in the earlier literature) defined for subsets of the set of integers, including frequent mean, frequent variance, frequent autocorrelation sequence, etc. These research may clarify several vague mathematical concepts and will form the basis for further research on sequences and discrete systems.

Keywords. Frequent mean, Frequent variance, Frequent autocorrelation sequence, Frequent chaos.

AMS (MOS) subject classification: 26A03, 28A12, 39A11, 40A05, 46A70.

1   Introduction

In [23], we introduced several new concepts for a real sequence such as frequent distribution, frequent density, integral, etc., and also studied some properties of these concepts. Based on these research, in this paper, we will continue to introduce several other new concepts for a sequence in order to study sequences and discrete systems better in future. For simplicity, in the following, all notations, concepts and theorems, etc., in [23] can be used and cited directly. For example, (7) in [23] denotes the set $\widetilde{\Omega}$ defined by (7) in the reference [23] which is not measurable, (16) in [23] denotes the cutoff sequence of a sequence X defined by (16) in [23], etc.

2   Frequent Mean of A Sequence

Definition 2.1. Let $X = \{x_n\}_{n=1}^{\infty}$ be a sequence. If the upper limit

$$\limsup_{n \to \infty} \frac{x_1 + x_2 + \cdots + x_n}{n} = \limsup_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} x_i$$