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## EXISTENCE AND GLOBAL ATTRACTIVITY OF POSITIVE PERIODIC SOLUTIONS IN A NONLINEAR DELAY POPULATION MODEL WITH IMPULSES

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Abstract. Consider the following nonlinear impulsive delay Nicholson's Blowflies model

 $\begin{cases} x'(t) = -\delta(t)x(t) + \sum_{i=1}^{n} p_i(t)x(t)e^{-\alpha_i(t)x(t-m_i\omega)}, \text{ a.e. } t > 0, \ t \neq t_k, \\ x(t_k^+) = (1+b_k)x(t_k), \quad k = 1, 2, \cdots, \end{cases}$ (1)

where  $m_i(i = 1, 2, \dots, n)$  is a positive integer,  $\delta(t), \alpha_i(t)$  and  $p_i(t)$  are positive periodic continuous functions with period  $\omega > 0$ . In the nondelay case (m = 0) we shall show that (1) has a unique positive periodic solution  $x^*(t)$ , and provide sufficient conditions for the global attractivity of  $x^*(t)$ . In the delay case we shall present sufficient conditions for permanence of the system (1), and establish sufficient conditions for the global attractivity of  $x^*(t)$ . It is shown that under appropriate linear periodic impulsive perturbations, the impulsive delay equation (1) preserves the original periodicity of the nonimpulsive delay equation, but the global attractivity of (1) depends on variation of the coefficient  $b_k$ . And our results improve some known conclusions.

**Keywords.** Positive periodic solution, global attractivity, impulsive, delay, Nicholson's blowflies model.

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## 1 Introduction

Impulsive differential equations play an important role in control theory, physics, chemistry, biology, population dynamics, biotechnologies, etc. For details, see [1] and [2]. Although impulsive perturbations make the systems more intractable, there are still some investigations, we refer to [9,17,18,20,23, 24,25] and references cited therein. And there is a tendency that more and more systems would be in the form of impulsive differential equations. Recently, some impulsive functional differential equations has been studied by several authors [3,6,14,15,16]. And for the fundamental theory of the impulsive differential equations, we may refer to the monographs [5] and [12].