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OSCILLATION FOR A CLASS OF NONLINEAR NEUTRAL DIFFERENCE EQUATIONS

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Abstract. This paper is concerned with the nonlinear neutral difference equation with positive and negative coefficients

$$\Delta\left[x_n - c_n f\left(x_{n-r}\right)\right] + p_n g\left(x_{n-k}\right) - q_n g\left(x_{n-l}\right) = 0,$$

where $\{c_n\}, \{p_n\}, \{q_n\}$ are nonnegative real sequences, $r > 0$, $0 \leq l \leq k-1$ are integers. Sufficient conditions for this equation to be oscillatory are established.

Keywords. Difference equation, nonlinear, neutral, oscillation.
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1 Introduction

In this paper, we investigated the oscillatory behavior of all solutions of the following neutral difference equation with positive and negative coefficients

$$\Delta\left[x_n - c_n f\left(x_{n-r}\right)\right] + p_n g\left(x_{n-k}\right) - q_n g\left(x_{n-l}\right) = 0, \tag{1}$$

where $\{c_n\}, \{p_n\}, \{q_n\}$ are nonnegative real sequences, $r > 0$, $k \geq 0$, $l \geq 0$ are integers with $0 \leq l \leq k-1$.
We will also assume that f, g are real continuous functions defined on $\mathbb{R}$ such that $xf(x) > 0$, $xg(x) > 0$ for $x \neq 0$ and

$$M_1 \leq \frac{f(x)}{x} \leq M_2, \quad N_1 \leq \frac{g(x)}{x} \leq N_2$$

for $x \in \mathbb{R}$, where M_1, M_2, N_1 and N_2 are some fixed positive integers.

When f and g are the identity functions, equation (1) reduces to the so called linear equation

$$\Delta\left[x_n - c_n x_{n-r}\right] + p_n x_{n-k} - q_n x_{n-l} = 0, \tag{2}$$

which oscillatory and nonoscillatory behaviors of solutions of equation (2) has been studied by many authors [1-9]. To the best knowledge of the author, there are no results available in literature dealing with the oscillatory behavior of equation (1).

In this paper, our aim is to give some sufficient conditions for the oscillation of equation (1). Our results improve the known results in the literature.