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COMPETITIVE EXCLUSION PRINCIPLE IN A MODEL OF CHEMOSTAT WITH DELAYS*

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Abstract. This paper is devoted to the study of the global asymptotic behavior of a model of chemostat with an arbitrary number of competitors following a Monod law on their specific growth rate functions. The model incorporates discrete time delays in order to take into account the delay in the conversion of nutrient consumed to the viable biomass. In this context, we state sufficient conditions ensuring that the presence of these time delays do not alter the prediction of the competitive exclusion principle. Our analysis and proofs rely on the construction of a Lyapunov-Krasovskii functional.

Keywords. Chemostat, competition, stability, delay, Lyapunov–Krasovskii functional.

AMS (MOS) subject classification: 34D05, 34D20, 34D23, 37B25.

1 Introduction

The chemostat is a laboratory apparatus used for the continuous culture of microorganisms. It is a benchmark model in microbial ecology, used for experimentally reproducing a wide variety of systems ranging from lakes, waste-water treatment plants, to reactors for commercial production of substances by genetically altered organisms.

Roughly speaking, it is basically a culture vessel having an input aperture for the influx of sterile nutrient medium from a reservoir and an overflow aperture for the efflux of exhausted medium, living cells, and cellular debris. The device (and the term "chemostat") was invented by Novick and Szilard [15]; the "bactogene" is a virtually identical device developed independently and simultaneously by Monod [14]. See, for instance, [6], [16], [4] for more details on this apparatus.

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