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## L<sup>2</sup>-STABILIZABILITY CONDITIONS FOR A CLASS OF NONSTANDARD SINGULARLY PERTURBED FUNCTIONAL-DIFFERENTIAL SYSTEMS

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Abstract. A nonstandard singularly perturbed linear time-invariant controlled system with the general type of delays in the state variables is considered. The delays are of two kinds: nonsmall in the slow and small in the fast state variables. Two much simpler parameter-free subsystems (the slow and fast ones) are associated with this system. Since the original singularly perturbed system is nonstandard, the slow subsystem is a descriptor (functional-differential-algebraic) one. It is established in the paper that  $L^2$ -stabilizability of the slow and fast subsystems yields  $L^2$ -stabilizability of the original system for all sufficiently small values of the parameter of singular perturbation. An extension of this result to the case of delay in the control variable also is presented. The theoretical results are illustrated by examples.

**Keywords.** Functional-differential system, time delay system, general type delay, singular perturbation, descriptor system, stabilizability.

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## 1 Introduction

Singularly perturbed controlled systems were extensively investigated in the literature because of their considerable theoretical and practical implications. Mainly, the systems without delays were studied (see e.g. [4,5,14,35,36,47] and references therein). The systems with delays were studied much less (see e.g. [9,13,15,24-26,37,38,42,53,57] and references therein).

 $L^2$ -stabilizability of a controlled system is one of its basic properties. This property means the ability to provide any solution of the system to be square-integrable over the right-hand half of the time-axis by a proper choice of the control function square-integrable over the same time-interval.  $L^2$ stabilizability is necessary for the existence of a solution of an infinite horizon linear-quadratic optimal control problem. Conditions of  $L^2$ -stabilizability, as well as conditions of various other types of stabilizability, for linear timeinvariant controlled systems without and with delays were extensively studied in the literature (see e.g. [30,39,48-50,55,56] and references therein). These conditions can be directly applied to a singularly perturbed system for any