Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **16 (2009)** 327-335
 http://www.watam.org

THREE SOLUTIONS TO A DIRICHLET PROBLEM FOR ELLIPTIC EQUATIONS INVOLVING THE P-LAPLACIAN IN N-DIMENSIONAL CASE

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Abstract. In this paper, we establish the existence of three weak solutions to a Dirichlet problem involving the p-Laplacian. Our main tool is a recent three critical points theorem of B. Ricceri [On a three critical points theorem, Arch. Math. (Basel) 75 (2000) 220-226].
Keywords. Three solutions; Critical point; Multiplicity results; Dirichlet problem.
AMS (MOS) subject classification: 35J65; 34A15.

1 Introduction

In this work, we study the boundary value problem

$$\begin{cases} \Delta_p u - a(x)|u|^{p-2}u = \lambda f(x,u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where $\Delta_p u = \text{div}(|\nabla u|^{p-2}\nabla u)$ is the p-Laplacian operator, $\Omega \subset R^N (N \geq 2)$ is non-empty bounded open set with smooth boundary $\partial\Omega$, $p > N$, $\lambda > 0$, $f : \Omega \times R \to R$ is a Carat$\acute{e}$odory function and positive weight function $a(x) \in C(\overline{\Omega})$.

Precisely, we deal with the existence of an open interval $\Lambda \subseteq [0, +\infty[$ and a positive real number q, such that, for each $\lambda \in \Lambda$, problem (1) admits at least three weak solutions whose norms in $W_0^{1,p}(\Omega)$ are less than q.

Let us recall that a weak solution of problem (1) is any $u \in W_0^{1,p}(\Omega)$ such that

$$\int_\Omega (|\nabla u(x)|^{p-2}\nabla u(x)\nabla v(x))dx + \int_\Omega (a(x)|u(x)|^{p-2}u(x)v(x))dx$$
$$+\lambda \int_\Omega f(x,u(x))v(x)dx = 0, \quad \forall v \in W_0^{1,p}(\Omega).$$

Problems of the above type studied in these latest years and we refer to [1-4] and the references therein for more details.

For instance, in [2], using variational methods, the authors ensure the existence of a sequence of arbitrarily small positive solutions for problem

$$\begin{cases} \Delta_p u + \lambda f(x,u) = 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{2}$$