GLOBAL ATTRACTIVITY AND CONVERGENCE OF A DIFFERENCE EQUATION

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Abstract. In this paper, we study the global behavior of the higher difference equation

$$x_{n+1} = \frac{\beta x_{n-k+1} + \gamma x_{n-2k+1}}{A + C x_{n-2k+1}}, \quad n = 0, 1, 2, \ldots,$$

where the initial conditions $x_{-2k+1}, , \ldots, x_1, x_0$ are nonnegative, $k \in \{1, 2, 3, \ldots\}$, and the parameters β, γ, A, C are positive. Briefly we concentrate on invariant intervals, the nature of semicycles, and the global stability of the above mentioned equation.

Keywords. Local asymptotic stability, Invariant interval, Semicycle behavior, Global asymptotic stability.

AMS (MOS) subject classification: 37B40

1   Introduction

The present paper is concerned with the quantitative and qualitative behavior of solutions to the difference equation

$$x_{n+1} = \frac{\beta x_{n-k+1} + \gamma x_{n-2k+1}}{A + C x_{n-2k+1}}, \quad n = 0, 1, 2, \ldots, \tag{1.1}$$

where the initial conditions $x_{-2k+1}, , \ldots, x_1, x_0$ are nonnegative, $k \in \{1, 2, 3, \ldots\}$, and the parameters β, γ, A, C are also positive.

Generally speaking, we first provide a simple change of variable in order to transform equation (1.1) to a new one that is easy during investigating its characteristics. We also show that zero is always an equilibrium point which satisfies a necessary and sufficient condition for its local asymptotic stability. Furthermore, it has another unique positive equilibrium point with a specific assumption on the parameters of the new equation with a condition for the local asymptotic stability of equation (1.1). Second we concentrate on analyzing the semicycles of solutions as well as discussing invariant intervals. In addition, we investigate the global stability of positive equilibrium point.

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