HILLE AND NEHARI TYPES OSCILLATION CRITERIA FOR SECOND-ORDER NEUTRAL DELAY DYNAMIC EQUATIONS

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Abstract. In this paper, we will establish some new oscillation criteria of Hille and Nehari types for the second-order neutral delay dynamic equation

$$[y(t) + r(t)y(\tau(t))]^{\Delta\Delta} + p(t)y(\delta(t)) = 0,$$

on a time scale $\mathbb{T}$. Our results in the special case can be applied on neutral delay differential and difference equations, i.e., when $\mathbb{T} = \mathbb{R}$ and $\mathbb{T} = \mathbb{N}$ and also can be applied when $\mathbb{T} = h\mathbb{N}$, for $h > 0$ and $\mathbb{T} = \mathbb{T}_n = \{t_n : n \in \mathbb{N}_0\}$ where $\{t_n\}$ is the set of the harmonic numbers defined by $t_0 = 0$, $t_n = \sum_{k=1}^{n} \frac{1}{k}$ for $n \in \mathbb{N}_0$. The main results include and improve some well-known oscillation results in the literature. An example is considered to illustrate the main results.

Keywords. *Oscillation, neutral delay dynamic equation, time scales.*

AMS (MOS) subject classification: *34B10, 39A10, 34K11, 34C10.*

1 Introduction

In this paper, we are concerned with oscillation of the second-order linear neutral delay dynamic equation

$$[y(t) + r(t)y(\tau(t))]^{\Delta\Delta} + p(t)y(\delta(t)) = 0, \tag{1.1}$$

on a time scale $\mathbb{T}$. Throughout this paper we assume that:

(h_1). τ and δ are positive $rd-$continuous functions defined on $\mathbb{T}$, $\tau(t) \leq t$, $\delta(t) \leq t$ and $\lim_{t\to\infty} \tau(t) = \lim_{t\to\infty} \delta(t) = \infty$,

(h_2). $r(t)$ and $p(t)$ are positive $rd-$continuous functions defined on $\mathbb{T}$ and $0 \leq r(t) < 1$.

Recall that a solution of (1.1) is a nontrivial real function $y(t)$ such that $y(t) + r(t)y(\tau(t)) \in C_{rd}^2[t_y, \infty)$ for $t_y \geq t_0$ and satisfying equation (1.1) for

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