Dynamics of Continuous, Discrete and Impulsive Systems Series B: Applications & Algorithms 16 (2009) 631-654 Copyright ©2009 Watam Press

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## INVARIANCE PRINCIPLES FOR IMPULSIVE SWITCHED SYSTEMS

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**Abstract.** The classical LaSalle's invariance principle is extended to a large class of impulsive switched systems. By introducing the notion of persistent limit set and persistent mode, some weak invariance principles are established, which generalize some known results to impulsive switched systems under weak dwell-time switchings. These weak invariance principles are then applied to derive several asymptotic stability criteria for impulsive switched systems. It is shown, by several examples, that even special cases of our results improve some known results in the literature.

**Keywords.** Switched systems; Impulsive Systems; Invariance principles; Stability; Lyapunov method; Multiple Lyapunov functions; Hybrid systems.

AMS (MOS) subject classification: 93B12, 93D20.

## 1 Introduction

Recently, the classical LaSalle's invariance principle [16, 17] has been extended to hybrid and switched systems by various authors (see, e.g., [1, 5, 7, 9,10,21,26]). In [9], under rather general switchings (including weak dwelltime switchings), an extension of LaSalle's principle is obtained for switched linear systems. In [1], a more traditional approach is taken and the results there cover general switched nonlinear systems, while a positive dwell-time condition is assumed. In [21], the results in [1,9] are extended and improved such that the results can deal with switched nonlinear systems with average dwell-time switching. Moreover, the weak invariance notion, which is essential to develop invariance principles for switched systems, is different from that of [1] and a more comprehensive property of the limit sets of a switched system is proved (Proposition 4.1 in [21]). The weak invariance principle improves the results in [1] (see how  $\mathcal{T}_{V}^{*}$  plays a role in the argument of Example 1 in [21]). However, the results in [21] still impose a moderate restriction on the switching rules, i.e., the average dwell-time restriction. More recently, the work of [26] investigates asymptotic stability of switched linear systems with dwell-time switchings using invariance-like ideas. It is shown