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HOPF BIFURCATION FOR A 3D FILIPPOV SYSTEM

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Abstract. We study the behaviour of solutions for a 3-dimensional system of differential equations with discontinuous right hand side in the neighbourhood of the origin. Using B-equivalence of that system to an impulsive differential equation [3, 4], existence of a center manifold is proved, and then a Hopf bifurcation theorem is provided for such equations in the critical case. The results are apparently obtained for the systems with dimensions greater than two for the first time. Finally, an appropriate example is given to illustrate our results.

Keywords. Filippov systems, Impulsive differential equations, Center manifold, Hopf bifurcation.

AMS (MOS) subject classification: 34A36, 34A37, 34C40, 37G15, 34K18, 34K19.

1 Introduction

When we consider bifurcations of a given type in a neighborhood of the origin, the center manifold theory appears as one of the most effective tools in the investigation. The study of center manifolds can be traced back to the works of Pliss [19] and Kelley [11]. When such manifolds exist, the investigation of local behaviours can be reduced to the study of the systems on the center manifolds. Any bifurcations which occur in the neighborhood of the origin on the center manifold are guaranteed to occur in the full nonlinear system as well. In particular, if a limit cycle exists on the center manifold, then it will also appear in the full system.

Physical phenomena are often modeled by discontinuous dynamical systems which switch between different vector fields in different modes. Filippov systems form a subclass of discontinuous systems described by differential equations with a discontinuous right-hand side [9]. Bifurcations in smooth systems are well understood, but little is known in discontinuous dynamical systems. In the last several decades, existence of non-smooth dynamics in the real world has stimulated the study of bifurcation of periodic solutions in discontinuous systems [8, 10], [12] – [18]. Furthermore, Bautin and Leontovich