Dynamics of Continuous, Discrete and Impulsive Systems
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DYNAMICAL SYSTEMS WITH DISCONTINUITIES

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Abstract. We survey several results on discontinuous systems. Then we present bifurcations of homoclinic and chaotic oscillations in differential equations with discontinuities. Finally we deal with the dynamics of epsilon inflated mappings and differential equations.

Keywords. Discontinuous systems, bifurcations, periodics, homoclinics, chaotic behavior, epsilon inflation.

AMS (MOS) subject classification: 34A36, 34A37, 34C15, 37G20.

1 Introduction

The purpose of this paper is to survey several results from the dynamics of discontinuous systems (DSs). We deal with discontinuous differential equations (DDEs), differential and difference inclusions as well. The plan of this paper is as follows: First we mention some general concepts of DSs with the corresponding literature. Then we illustrate bifurcations of homoclinic and chaotic solutions for impulsive differential equations (IDEs), differential inclusions (DIs) and weakly discontinuous differential equations (WDDEs). We also consider cases when homoclinic solutions either slide on surfaces of discontinuity or transversally cross them. The final part of the paper is devoted to epsilon inflated dynamical systems which naturally appear in set valued approximations arising in computer-assisted proofs.

1.1 Discontinuous Dynamical Systems

Dynamical systems with non-smooth components appear in various problems in physics, biology, economics, chemistry and control theory as well. A nice introduction to DSs is given in [30, 38, 49, 52, 63]. DSs include many different discontinuous phenomena like DDEs, IDEs, DIs, systems with impacts or with hysteresis behavior. The theory of smooth dynamical systems is rather well developed [57, 81], but new interesting discontinuous models force nowadays to establish similar theory for DSs. We present several results in Section 1.2 to justify this rapid development of DSs.