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EFFECT OF THE DIFFUSION COEFFICIENT TO THE COST OF CONTROLS

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Abstract. We study the estimates of the cost of approximate controllability and null controllability of parabolic equations in divergence form. Especially, the explicit bound of the cost with respect to the diffusion coefficient $a(x,t)$ and the time T is analyzed. The methods of proof combines global Carleman inequality, energy estimates for parabolic equations and the variational approach to controllability.

Keywords. Approximate controllability, null controllability, cost of the controllability, Carleman inequality, observability inequality.

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1 Introduction

Let Ω be a bounded domain of $\mathbb{R}^N$ with boundary of class C^2 and ω be a non-empty open set of Ω. Consider the following control system:

$$\begin{cases} u_t - \operatorname{div}(a(x,t)\nabla u) = f\chi_\omega & \text{in } Q_T, \\ u = 0 & \text{on } \Sigma_T, \\ u(x,0) = u_0(x) & \text{in } \Omega, \end{cases} \tag{1}$$

where $Q_T = \Omega \times (0,T)$, $\Sigma_T = \partial\Omega \times (0,T)$, $u = u(x,t)$ is the state, $f = f(x,t)$ is the control, and χ_ω denotes the characteristic function of the set ω.

We say that system (1) is *approximately controllable* in $L^2(\Omega)$ at time $T > 0$, if for every $u_0 \in L^2(\Omega)$, every final state $u_d \in L^2(\Omega)$ and $\varepsilon > 0$, there exists a control $f \in L^2(Q_T))$ such that the solution of (1) satisfies

$$\|u(x,T) - u_d\|_{L^2(\Omega)} \leq \varepsilon. \tag{2}$$

In general, approximate controllability can be viewed as a consequence of the null controllability property of the linear parabolic system (1). We recall that system (1) is said to be *null controllable* if, for every $u_0 \in L^2(\Omega)$, there exists a control $f \in L^2(Q_T)$ such that the solution of (1) satisfies

$$u(x,T) = 0 \qquad \text{a. e. in } \Omega. \tag{3}$$