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EXISTENCE OF PERIODIC SOLUTIONS OF NONLINEAR DELAY DIFFERENTIAL EQUATIONS WITH IMPULSES

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Abstract. Existence of periodic solutions of nonlinear and nonautonomous impulsive differential equation with multiple variable lags is proved by using theory of coincidence degree. An example is provided to illustrate the theory.

Keywords. Periodic solution, Nonlinear, Functional differential equation, Impulse, Coincidence degree.

AMS (MOS) subject classification: 34K15, 34C25

1 Introduction

The theory of impulsive delay differential equations is emerging as an important area of investigation since it is a lot richer than the corresponding theory of nonimpulsive delay differential equations. Many evolutionary processes in nature are characterized by the fact that at certain moments in time an abrupt change of state is experienced. That is the reason for the rapid development of the theory of impulsive differential equations and impulsive delay differential equations [1, 2, 6, 10]. Because the existence of periodic solutions of impulsive functional differential equation has many applications in population dynamics, biology, biotechnology, etc., it has been becoming a focus of research in functional differential equation. In recent years, many researchers have done some significant studies in this field [4, 7, 11, 12].

In [5], Kocic, Ladas and Qian investigated the oscillatory behavior of the following quite general nonlinear and nonautonomous delay differential equation:

$$\dot{x}(t) + f(t, x(t-\tau_1(t)), x(t-\tau_2(t)), \cdots, x(t-\tau_m(t))) = 0,$$

where

$$\begin{cases} f \in C[[t_0,\infty) \times R^m, R],\ f(t,u_1,\cdots,u_m) \ge 0 \text{ for } u_1,\cdots,u_m \ge 0, \\ f(t,u_1,\cdots,u_m) \le 0 \text{ for } u_1,\cdots,u_m \le 0,\ \text{and for }\ i=1,\cdots,m, \\ \tau_i \in C[[t_0,\infty),(0,\infty)] \text{ and } \lim_{t\to\infty}[t-\tau_i(t)] = \infty. \end{cases}$$