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A DISCRETE-TIME LYAPUNOV THEOREM FOR THE EXPONENTIAL INSTABILITY OF C_0-SEMIGROUPS

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Abstract. A discrete-time Lyapunov approach is proposed to study the exponential instability of C_0 -semigroups. Also, some continuous-time Lyapunov conditions are established.

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1 Introduction

Let X be a Hilbert space and $B(X)$ the Banach algebra of all bounded linear operators acting on X. Consider the Abstract Cauchy Problem

$$\frac{du(t,x)}{dt} = Au(t,x),\ u(0,x) = x \in X,\ t \geq 0\ (ACP)$$

with a closed densely defined (possible unbounded) linear operator A.

Assume for a moment that (ACP) is well-posed; that is A generates a C_0-semigroup $\mathbf{T} = T(t)_{t\geq 0}$ on X. We recall that a $B(X)$-valued function $\mathbf{T} = \{T(t)\}_{t\geq 0}$ is named a semigroup of linear operators if the identity on X can be obtained as $T(0)$ and the semigroup property $T(t+s) = T(t)T(s)$ is also satisfied for all $t, s \geq 0$. If in addition $\mathbf{T}$ is strongly continuous (i.e. there exists $\lim\limits_{t\to 0_+} T(t)x = x$, for all $x \in X$) then we will call $\mathbf{T}$ as a C_0-semigroup. It is widely known that each C_0-semigroup is exponentially bounded i.e.

$$\|T(t)\| \leq Me^{\omega t},\ \text{for all } t \geq 0$$

for some $M, \omega > 0$ (see for instance [6, 11]). Also we will denote by

$$D(A) = \left\{ x \in X :\ \text{there exists } \lim_{t\to 0+} \frac{T(t)x - x}{t} \right\},\ Ax = \lim_{t\to 0+} \frac{T(t)x - x}{t}.$$

The operator A is called the infinitesimal generator of $\mathbf{T}$ and is proved to be closed and densely defined. Also it is known that if $x \in D(A)$ then $T(\cdot)x$ is continuously differentiable with