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SMOOTHNESS OF DENSITY FUNCTION FOR RANDOM MAPS

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Abstract. A discrete-time random dynamical system is said to be a random map if one of a number of transformations is randomly selected and applied at each iteration of the process. Invariant densities of random maps describe the asymptotic properties of a random map. If the individual maps of a random map are piecewise onto and piecewise expanding then the random map satisfies Pelikan's average expanding condition and the random map has invariant densities. For individual maps, piecewise expanding and piecewise onto are sufficient to establish many important properties of the invariant densities, in particular, the fact that the densities inherit smoothness properties of individual maps. It is of interest to see if this property is transferred to random maps satisfying piecewise expanding and piecewise onto conditions. We show that if all the maps constituting the random map are piecewise expanding and piecewise onto, then the same result is true.

Keywords. Random maps; Frobenius-Perron operator; Density function; Smoothness; Absolutely continuous invariant measures.

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1 Introduction

One of the fundamental problems in ergodic theory is to describe the asymptotic behavior of trajectories defined by a dynamical system. The existence and properties of invariant measures for chaotic dynamical systems reflect their long time behavior and play an important role in understanding their chaotic nature. For a single transformation of an interval, much is known about the densities of the absolutely continuous invariant measures (acim). For example, it is known that the densities inherit smoothness properties from the map itself (Halfant [8] , Szewc [16]), that the supports consist of a finite union of intervals and that the densities are bounded below on their supports (Keller [9] and Kowalski [10]).

Random dynamical systems provide a useful framework for modeling and analyzing various physical, social and economic phenomena [5,14]. A random dynamical system of special interest is a random map where the process