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A NOTE ON BIFURCATION STRUCTURE OF RADIALLY SYMMETRIC STATIONARY SOLUTIONS FOR A REACTION-DIFFUSION SYSTEM II

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Abstract. In this paper, we discuss the local structure of radially symmetric stationary solutions for a certain reaction-diffusion system. To do it, we employ the comparison principle and the numerical verification method, and estimate the integral for the Bessel function of the first kind which is an eigenfunction of the linearized operator around the constant stationary solution.

Keywords. bifurcation structure, interval arithmetic, Bessel function.

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1 Introduction

To understand the mechanism of phenomena which appear in various fields, we often employ the system of reaction-diffusion equations

$$\begin{cases} \frac{\partial}{\partial t}\mathbf{u} = \varepsilon\, D\, \Delta\mathbf{u} + \mathbf{f}(\mathbf{u}), & x \in \Omega, \qquad t > 0, \\ \frac{\partial}{\partial \nu}\mathbf{u} = \mathbf{0}, & x \in \partial\Omega, \qquad t > 0 \end{cases} \tag{1.1}$$

with suitable initial condition, and discuss the existence and stability of stationary solutions for the system, where $\mathbf{u} \in \mathbb{R}^N$, $\varepsilon > 0$, D is an $N \times N$ diagonal matrix whose elements are positive, $\mathbf{f} : \mathbb{R}^N \to \mathbb{R}^N$ is a smooth function in $\mathbf{u}$, Ω is a bounded domain in $\mathbb{R}^\ell$ with smooth boundary $\partial\Omega$, and $\frac{\partial}{\partial\nu}$ denotes the outward normal derivative on $\partial\Omega$.

When $N = 1$ is satisfied, it is well-known that for suitable $\mathbf{f}(\mathbf{u})$, the global attractor $\mathcal{A}$ of (1.1) is represented as $\mathcal{A} = \bigcup_{\mathbf{e}\in E} W^u(\mathbf{e})$, where E is the set of stationary solutions for (1.1), and $W^u(\mathbf{e})$ is an unstable manifold of (1.1) at $\mathbf{u} = \mathbf{e}$ (for example, see Chapter 4 in Hale [2]). This result suggests that we need to seek out all stationary solutions of (1.1), to understand the precise asymptotic behavior of solutions for (1.1). Along this line, in this paper, we assume that there exists a solution $\hat{\mathbf{u}} \in \mathbb{R}^N$ of $\mathbf{f}(\mathbf{u}) = \mathbf{0}$ with $(-1)^N \det \mathbf{f}_{\mathbf{u}}(\mathbf{u}) < 0$, and we discuss the bifurcation structure of radially symmetric stationary solutions for (1.1) with $\Omega = \left\{\, x \in \mathbb{R}^\ell \,\middle|\, |x| < \pi \,\right\}$.