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THE ECKSTEIN-BERTSEKAS PROXIMAL POINT ALGORITHM AND ITS APPLICATIONS TO INCLUSION PROBLEMS BASED ON THE GENERALIZED FIRM NONEXPANSIVENESS

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Abstract. Based on the generalized firm nonexpansiveness of the resolvent operator, the convergence analysis of the Eckstein-Bertsekas proximal point algorithm in the context of approximating the solution of a class of nonlinear inclusion problems is given. Several new results on the generalized firm nonexpansiveness of the resolvent operator are also established. The obtained results are general in nature, and can be applied to the generalized Douglas-Rachford splitting method.
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1 Introduction

We consider the problem of determining a solution to the inclusion

$$0 \in M(x), \tag{1}$$

where X is a real Hilbert space with the norm $\|.\|$ and the inner product $\langle .,. \rangle$, and $M : X \to 2^X$ is a set-valued mapping on X.

Rockafellar [22] generalized the algorithm of Martinet [13] for convex programming, which is referred to as the proximal point algorithm in literature. Then Rockafellar [23] examined its general convergence and rate of convergence analysis in the context of solving (1) with M monotone, and has further established that when M is maximal monotone, the sequence $\{x^k\}$ generated for an initial point x^0 by

$$x^{k+1} \approx P_k(x^k), \tag{2}$$

converges weakly to a solution to (1), provided the approximation is made sufficiently accurate as the iteration proceeds, where $P_k = (I + c_k M)^{-1}$ for a sequence $\{c_k\}$ of positive real numbers that are bounded away from zero. It follows from (2) that x^{k+1} is an approximate solution to the inclusion

$$0 \in M(x) + c_k^{-1}(x - x^k). \tag{3}$$

The general theory of multivalued maximal monotone mappings provides a general framework to studying convex programming and variational inequality problems based on algorithms such as the generalized alternating