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## ESTIMATION OF REGIONS OF ASYMPTOTIC STABILITY OF NONLINEAR POLYNOMIAL SYSTEMS

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**Abstract.** This paper firstly presents the estimation on the region of asymptotic stability (RAS) of nonlinear polynomial systems with linear and quadratic or linear and cubic forms for engineering applications. The estimations on RAS are then discussed for general nonlinear systems with linear, quadratic, cubic, and other forms. Using Lyapunov function approach, a closed-form estimation on RAS is derived in terms of quadratic functions. Computer simulation is provided to illustrate the new results of the proposed method.

**Keywords.** Nonlinear systems, regions of asymptotical stability, Lyapunov function methods, nonlinear polynomial systems, simulation.

AMS (MOS) subject classification: 37C75, 65L99, 93D20, 93D30, 94C99

## 1 Introduction

Consider a continuous-time nonlinear circuit or power system

$$\dot{x}(t) = f(x(t)), \quad t \in R, \quad x(t) \in R^n,$$

where f(x) is an analytic vector field. Let  $x_e$  be an equilibrium point and  $x(t, x_0)$  be the solution at the point  $x(t_0, x_0) = x_0$  of the system. The stable and unstable sub-manifolds of the equilibrium point  $x_e$  are defined as

$$W^{s}(x_{e}) = \{x_{0} \in \mathbb{R}^{n} : \lim_{t \to +\infty} x(t, x_{0}) = x_{e}\},\$$
$$W^{u}(x_{e}) = \{x_{0} \in \mathbb{R}^{n} : \lim_{t \to -\infty} x(t, x_{0}) = x_{e}\}.$$

Further, the region of asymptotical stability (RAS) is defined as  $W^s(x_e)$ , whose boundary is often denoted by  $\partial W^s(x_e)$ .

An equilibrium point  $x_e$  is *hyperbolic* if the Jacobian matrix of the function f at  $x_e$ , denoted by  $J_f(x_e)$ , has no eigenvalues with zero real part. A hyperbolic equilibrium point  $x_e$  is said to be of type-k if  $J_f(x_e)$  has k positive real part eigenvalues.

Determining the RAS or the domain of attraction (DA) of a stable equilibrium point for a high-order nonlinear autonomous system is important in many engineering applications<sup>[1-4]</sup>, such as electronic systems, economics,