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POSITIVE SOLUTIONS TO SECOND ORDER FOUR–POINT IMPULSIVE PROBLEMS WITH DEVIATING ARGUMENTS

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Abstract. In this paper we discuss four–point boundary value problems for second impulsive differential equations with deviating arguments. Deviating arguments can be of delayed or advanced types. Sufficient conditions are formulated under which such problems have at least three positive solutions. To obtain existence results we use the Leggett–Williams theorem.

Keywords: Impulsive differential equations, deviating arguments, existence of positive solutions.
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1 Introduction

For $J = [0, 1]$, let $0 = t_0 < t_1 < \cdots < t_m < t_{m+1} = 1$. Put $J' = (0, 1) \setminus \{t_1, t_2, \cdots, t_m\}$. Put $\mathbb{R}_+ = [0, \infty)$ and $J_k = (t_k, t_{k+1}],\ k = 0, 1, \cdots, m-1,\ J_m = (t_m, t_{m+1})$.

Let us consider second order impulsive differential equations of type

$$\begin{cases} x''(t) + h(t)f(x(\alpha(t))) = 0, \quad t \in J', \\ \Delta x'(t_k) = Q_k(x(t_k)), \quad k = 1, 2, \cdots, m, \\ x(0) = \gamma x(\xi), \quad \beta x(\eta) = x(1), \end{cases} \tag{1}$$

where as usual $\Delta x'(t_k) = x'(t_k^+) - x'(t_k^-)$, $x'(t_k^+)$ and $x'(t_k^-)$ denote the right and left limits of x' at t_k, respectively.

We assume that:

H_1 : $f \in C(\mathbb{R}_+, \mathbb{R}_+),\ \alpha \in C(J, (0, 1]),\ t \le \alpha(t) \le 1$ for $t \in J$ and if there exists a point $\bar{t} \in J$ such that $\alpha(\bar{t}) \in \{t_1, t_2, \cdots, t_m\}$, then $\bar{t} \in \{t_1, t_2, \cdots, t_m\}$,

H_1' : $f \in C(\mathbb{R}_+, \mathbb{R}_+),\ \alpha \in C(J, (0, 1]),\ \alpha(t) \le t$ for $t \in J$ and if there exists a point $\bar{t} \in J$ such that $\alpha(\bar{t}) \in \{t_1, t_2, \cdots, t_m\}$, then $\bar{t} \in \{t_1, t_2, \cdots, t_m\}$,

H_2 : $h \in C(J, \mathbb{R}_+)$ and h does not vanish identically on any subinterval, $Q_k \in C(\mathbb{R}_+, (-\infty, 0])$ and are bounded for $k = 1, 2, \cdots, m$,