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SUCCESSIVE ITERATION AND POSITIVE SOLUTIONS FOR SOME SECOND-ORDER IMPULSIVE MULTI-POINT BOUNDARY VALUE PROBLEM ON HALF-LINE

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Abstract. This paper deals with the existence of positive solutions for some second-order impulsive multi-point boundary value problem on the half-line. Our approach is based on the fixed point theorem and the monotone iterative technique. Without the assumption of the existence of lower and upper solution, we obtain not only the existence of positive solutions for the problems, but also establish iterative schemes for approximating the solutions.

Keywords. Successive iteration; Singular differential equation; Positive solutions; Half line; Fixed point theorem.

AMS (MOS) subject classification: 34B10, 34B15, 34B18, 34B40.

1 Introduction

In this paper, we are concerned with the positive solutions to the following second-order multi-point boundary value problem

$$\begin{cases} x''(t) + q(t)f(t, x(t)) = 0, \quad t \in J'_+, \\ \Delta x|_{t=t_k} = I_k(x(t_k)), \ k = 1, 2, 3, \cdots, n, \\ x(0) = \sum_{i=1}^{m-2} \alpha_i x(\xi_i), \quad x'(\infty) = x_\infty \geq 0, \end{cases} \tag{1}$$

where $J = [0, +\infty), J_+ = (0, +\infty), J'_+ = J_+ \backslash \{t_1, \cdots, t_n\}, \alpha_i \in J, t_k$ $(k = 1, 2, \cdots, n,$ where n is a fixed positive integer) are fixed points with $0 < t_1 < \cdots < t_k < \cdots < t_n < +\infty, \xi_i \in J_+$ with $0 < \xi_1 < \xi_2 < \cdots < \xi_{m-2} < +\infty, \xi_i \neq t_k$ $(i = 1, 2, \cdots, m-2, \ k = 1, 2, \cdots, n)$, $0 < \sum_{i=1}^{m-2} \alpha_i < 1, x'(\infty) = \lim\limits_{t \to +\infty} x'(t)$, $\Delta x|_{t=t_k}$ denotes the jump of $x(t)$, i.e.,

$$\Delta x|_{t=t_k} = x(t_k^+) - x(t_k^-),$$

where $x(t_k^+)$ and $x(t_k^-)$ represent the right-hand limit and left-hand limit of $x(t)$ at $t = t_k$, respectively. Throughout this paper, we always assume the following conditions are satisfied.