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HIERARCHICAL SIZE-STRUCTURED POPULATIONS: THE LINEARIZED SEMIGROUP APPROACH

József Z. Farkas[1] and Thomas Hagen[2]

[1]Department of Computing Science and Mathematics
University of Stirling, Stirling, FK9 4LA, UK

[2] Department of Mathematical Sciences
The University of Memphis, Memphis, TN 38152, USA

Corresponding author email:jzf@maths.stir.ac.uk

Abstract. In the present paper we analyze the linear stability of a hierarchical size-structured population model where the vital rates (mortality, fertility and growth rate) depend both on size and a general functional of the population density ("environment"). We derive regularity properties of the governing linear semigroup, implying that linear stability is governed by a dominant real eigenvalue of the semigroup generator, which arises as a zero of an associated characteristic function. In the special case where neither the growth rate nor the mortality depend on the environment, we explicitly calculate the characteristic function and use it to formulate simple conditions for the linear stability of population equilibria. In the general case we derive a dissipativity condition for the linear semigroup, thereby characterizing exponential stability of the steady state.

Keywords. Hierarchical size-structured populations; Semigroup methods; Spectral analysis; Principle of linear stability.

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1 Introduction

In the last three decades nonlinear age- and size-structured population models have attracted a lot of interest both among theoretical biologists and applied mathematicians. Traditionally, structured population models have been formulated as partial differential equations for population densities. Starting with the seminal work [16], researchers have been developing and analyzing various physiologically structured population models. We refer here to the well-known monographs [5, 19, 22, 25].

Diekmann et al. have been developing a general mathematical framework for modeling structured populations, see for example [9, 10]. One of their most important recent results is that the qualitative behavior of nonlinear physiologically structured population models can be studied by means of linearization [7, 8]. In other words, they have proven for a very general