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EXISTENCE OF SOLUTIONS FOR NONLINEAR 2ND-ORDER IMPULSIVE INTEGRO-DIFFERENTIAL EQUATIONS OF MIXED TYPE IN BANACH SPACE

Xiaoyan Zhang

School of Mathematics
Shandong University, Jinan, Shandong 250100, P. R. China
Corresponding author email: zxysd@mail.sdu.edu.cn

Abstract. In this paper, under rather weak conditions, we establish the new existence results of solutions for the initial value problem of nonlinear second-order impulsive integro-differential equations of mixed type in Banach spaces by reducing the order of equation and using Mönch fixed point theorem on each interval. Our results improve and unify various recent results.

Keywords. Banach Space, measure of noncompactness, second order, impulsive integro-differential equation, Mönch fixed point theorem.

AMS (MOS) subject classification: 34K30, 45J05.

1 Introduction

Let $(E,\ \|\cdot\|)$ be a real Banach space, $J=[0,a]$ $(a>0)$, $0=t_0<t_1<t_2<\cdots<t_k<\cdots<t_m<t_{m+1}=a$, $f\in C(J\times E\times E\times E\times E,E)$, $I_k,\ \bar{I}_k\in C(E\times E,E)$ $(k=1,2,\cdots,m)$, $x_0,\ x_1\in E$.

In this paper, we shall investigate the following initial value problem (IVP) for nonlinear second-order impulsive integro-differential equations of mixed type in E:

$$\begin{cases} x''=f(t,x,x',Tx,Sx), \quad t\in J,\ \ t\neq t_k,\\ \triangle x|_{t=t_k}=I_k(x(t_k),x'(t_k)),\\ \triangle x'|_{t=t_k}=\bar{I}_k(x(t_k),x'(t_k)) \quad (k=1,2,\cdots,m),\\ x(0)=x_0,\ \ x'(0)=x_1, \end{cases} \tag{1.1}$$

in which

$$(Tx)(t)=\int_0^t k(t,s)x(s)ds,\ \ (Sx)(t)=\int_0^a h(t,s)x(s)ds,\ \ t\in J, \tag{1.2}$$

where $k\in C(D,R)$, $D=\{(t,s)\in J\times J:\ t\geq s\}$, and $h\in C(J\times J,R)$, R is the set of all real numbers. $\triangle x|_{t=t_k}$ denotes the jump of $x(t)$ at $t=t_k$, i.e.,