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## FURTHER EXTENSIONS ON A STABILITY PROPERTY WITH SLOWLY VARYING INPUTS

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Abstract. In this paper we present three extended results on the stability property with slowly varying inputs that has been studied in, e.g., Khalil and Kokotović (1991). The stability property has played an important role in control methodologies such as gainscheduling controls or robust controls for systems having slowly varying parameters. We begin with a review of the first extension, which shows that the inputs may vary fastly, but once its 'average' is slowly varying, then the same stability property still holds. The second extension deals with the case when a (transcritical) bifurcation occurs so that the assumptions of the stability property are violated. We propose a Jump & Wait strategy which will still guarantee the stability property even under this case. Finally we prove that the Jump & Wait strategy can be employed not with the input itself but with the average of the input if the proposed structural condition is satisfied.

**Keywords.** Slowly varying input; Averaging theory; Transcritical bifurcation; Positive system; Passing through bifurcation.

## 1 Introduction

In the early 1990s, Kelemen [8], Lawrence and Rugh [12], and Khalil and Kokotović [9] have presented a stability property of nonlinear systems with slowly varying inputs, which can now be found in a graduate textbook such as [10]. The result is stated as follows: consider a dynamic system given by

$$\dot{x} = f(x, u),\tag{1}$$

where  $f(\cdot, \cdot)$  is continuously differentiable,  $x \in \mathbb{R}^n$ , and  $u \in \Gamma \subset \mathbb{R}^m$  in which  $\Gamma$  is a connected compact set. Suppose that, for each frozen (i.e., constant) input  $u \in \Gamma$ , there exists a corresponding isolated equilibrium  $x^*(u)$  such that  $f(x^*(u), u) = 0$  and  $x^*(\cdot)$  is continuous and piecewise twice continuously differentiable. Then, the following theorem holds.

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