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APPROXIMATE CONTROLLABILITY OF INTEGRO-DIFFERENTIAL EQUATIONS IN A HILBERT SPACE WITH NONLOCAL CONDITIONS

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Abstract. In this paper we shall study an integro-differential equation in a separable Hilbert space. We shall use the compact semigroup theory of linear operators and Schauder fixed point theorem to study the approximate controllability of the given problem. We shall also study the complete controllability of the given system with the help of Banach fixed point theorem. Finally, we shall give some examples to illustrate the application of our results.

Keywords. Integro-differential equation; Approximate controllability; Complete controllability; Compact semigroup; Schauder fixed point theorem; Banach fixed point theorem.

AMS (MOS) subject classification: 34K35; 93B05; 93C25.

1 Introduction

We consider the following nonlocal evolution problem in a separable Hilbert space $(H, \|.\|, \langle .,. \rangle)$:

$$\begin{aligned} x(t) &= S(t)x(0) + \int_0^t S(t-s)[Bu(s) \\ &\quad + f(s, x(s), \int_0^s g(s, \eta, x(\eta))d\eta, \int_0^T h(s, \eta, x(\eta))d\eta)]ds, \ t \in J = [0, T], \\ x(0) &= x_0 + \phi(x), \end{aligned} \tag{1.1}$$

where $\{S(t) : t \geq 0\}$ is a strongly continuous semigroup generated by a linear operator, $x : [0, T] \to H$ is the state function, $u \in L^2(0, T; U)$ is the control function, U is a Hilbert space, $B : U \to H$ is a bounded linear operator and $\phi : C(0, T; H) \to H$ is a given function (will be specified later). The